

## EMERGENT COLLECTIVE BEHAVIOR: PROBING THE DENSITY DEPENDENCE WITH THE VICSEK MODEL

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### Abstract

The Vicsek model provides as a primary outline for studying the emergence of collective motion or “flocking” from simple limited interaction rules among self – propelled particles. This work examines the particular effect of particle density on the collective behaviour and commanding within such an organization. Simulations were carried out using FORTRAN program in a Linux atmosphere, with parameters including particle speed, noise and interaction radius held constant whereas carefully varying the number of particles (N) from 10 to 2000 within a fixed spatial box. The normalized average velocity ( $v_a$ ) was used for quantifying the degree of collective motion. The results reveal apparent and non-monotonic reliance behaviour on particle density. Particles move independently with minimal consistency at very low densities. By increasing density, transitions takes place where particles begin to form coherently moving clusters. Although, at intermediary densities, group formation can lead to a decrease in universal order if groups move in dissimilar directions. At extremely high densities, a high degree of global alignment is constantly covered, with the system possessing strong polar order ( $v_a \approx 0.999$ ). The present work concludes that while enhancing density usually promotes collective motion, the relation is complicated, with optimal global coordination emerging at sufficiently high densities where local interactions percolate throughout the complete system.

### Keywords:

*Vicsek model, particle density, collective behaviour, self-propelled particles, local clusters.*

## INTRODUCTION

Collective behaviour is everywhere occurrence in nature, observed across an extensive range of scales, from swirling flocks of birds and schools of fish to the movement of bacterial colonies and human crowds [1-3]. These arrangements are categorized by the emergence of large scale, coordinated motion from the interaction between many individuals, each following simple set of laws without centralized control. Considering the fundamental principles that govern this self – organization is an essential challenge in statistical physics and composite science [4-6].

To theoretically discover these happenings, Tamas Vicsek and his colleagues introduced a seminal model in 1995, now identified as the Vicsek Model [5-8]. This model presents a simple yet powerful framework for studying active matter. It involves self-propelled particles that move at a constant speed but their direction of motion depends on the average direction of their neighbours within a preset interaction radius, with adding of arbitrary noise term. Regardless of its minimalism, the Vicsek model shows a rich phase transition from disordered movement to the ordered collective motion, comparable to ferromagnetic systems, controlled by parameters like, particle density and noise [9].

Although the function of noise in disrupting order is well established, the power of particle density on the collective behaviour is equally significant and shows more nuanced dynamics [10-12]. The frequency and the range of interaction between particles is determined by density, basically shaping how information in this condition, directional alignment propagates through the system. It is hypothesized that at very low densities, particles are too light to sway each other, leading to a disordered state. Local clusters start to form as density increases, but global system may not obtain complete coherence if these clusters move independently. At a crucial density, a phase transition to a universal ordered state is supposed [13,14].

The main objective of this research is to thoroughly investigate this effect of particle density on the collective dynamics within the Vicsek model. We aim to map out the relationship between density and the resulting order, by holding other parameters such as, noise, speed, and system size and changing only the number of particles. We quantify the collective motion through the normalized average velocity and determine the formation and interaction of particle clusters using numerical simulations. This work seeks to elucidate how density drives the transition from independent motion to local clustering and, ultimately, to global polar order, providing deeper insight into the self-organization principles of active matter.

## Methodology

The Vicsek model was utilised in this study to achieve its goals.

Position of  $i$ th particle updated according to

$$x_i(t + 1) = x_i(t) + v_i(t)\Delta t \dots\dots\dots (1)$$

In this case, the direction is determined by the angle  $\theta(t + 1)$ , and the velocity of a particle  $V_i(t + 1)$  was designed to have an absolute value  $v$ . The expression yielded this angle.

$$\theta(t+1) = \langle \theta(t) \rangle r + \Delta\theta \dots \dots \dots (2)$$

The average direction of the velocities of the particles (including the particles  $i$ ) within a circle of radius  $r$  encircling the given particle is indicated by  $\langle \theta(t) \rangle r$ .

The angle  $\arctan[\langle \sin(\theta(t)) \rangle / \langle \cos(\theta(t)) \rangle]$  provided the average direction. The random number  $\Delta\theta$  in Eq. (2) is selected uniformly from the interval  $[-n/2, n/2]$ . Therefore, the phrase stands for noise, which we will employ as a variable similar to temperature. Accordingly, for a given system size, there are three free parameters:  $n$ ,  $p$ , and  $v$ , where  $v$  is the particle's distance between two updates.

The following formula determines how the particles behave collectively.

$$v_a = \frac{1}{Nv} \left| \sum_{i=1}^N v_i \right| \dots \dots \dots (3)$$

Where  $N$  is the particle count and  $v$  is the velocity. FORTRAN programming has been utilized for the simulation. GNU Plot has been utilized to visualize the data. The software is run on the Linux operating system.

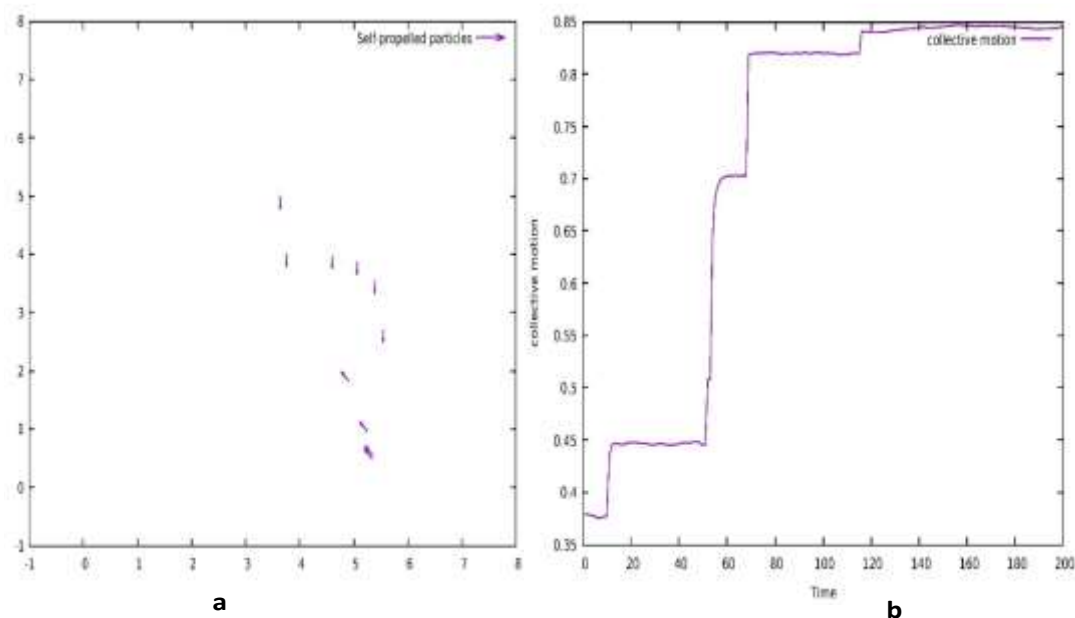
## Results and Discussion

### Parameter Table

Table 4.1 lists the parameters utilized in the Vicsek model, which include particle density, particle speed, particle noise, and particle interaction.

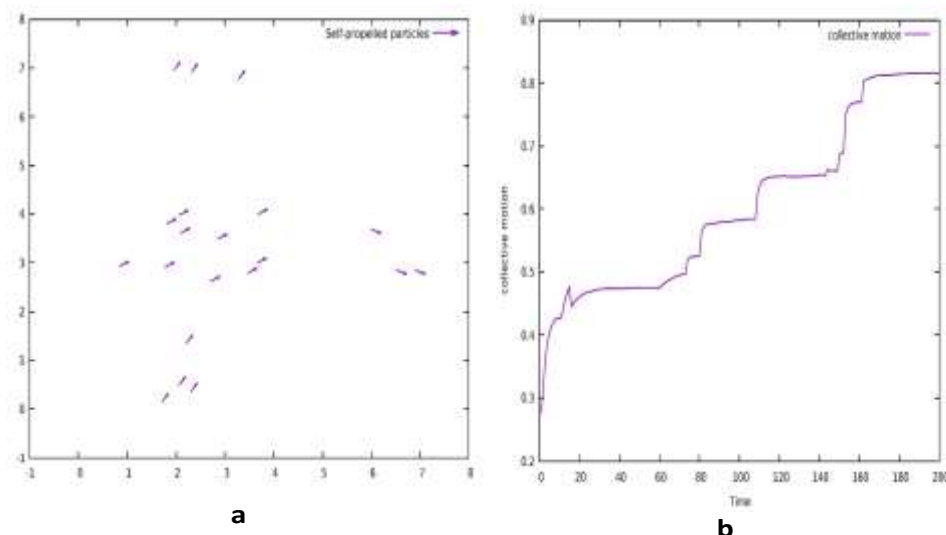
**Table 4.1 Symbols used in the Method of Simulation**

S. No	Description	Symbol
1	Interactive radius	I
2	Noise	H
3	Number of time steps	T
4	Absolute Velocity or speed	v
5	Interaction radius	R
6	Box Length	L
7	Number of Particles	N



**Figure 1** There is a certain correlation in the framework ( $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 10$ ,  $R = 0.919$ ).

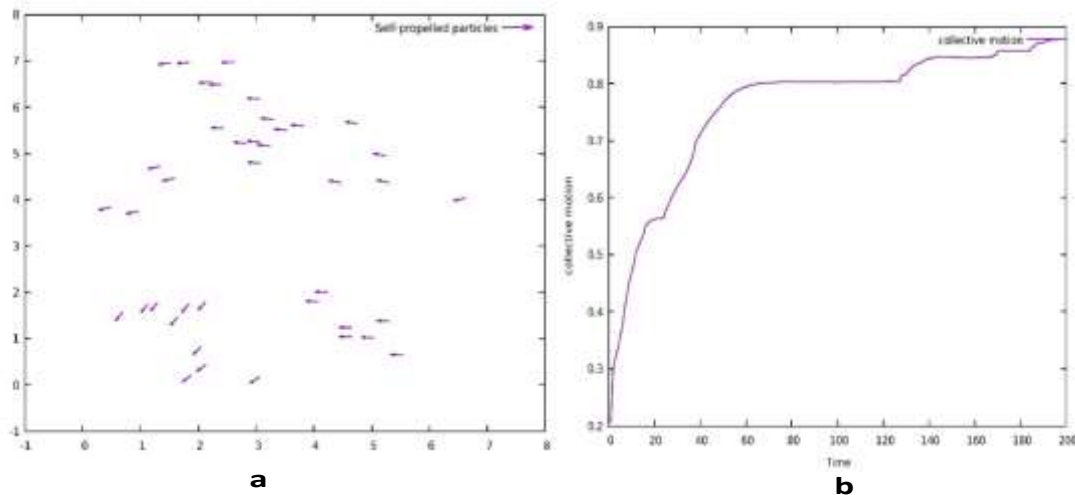
Only the number of particles is increased from 5 to 10 in figure 2, but all other parameters remain the same as in Figure 1. There is minimal variation in the particle interaction since the other parameters remain constant. Although they have become closer, there has been no discernible impact on their coherence. With  $R = 0.919$ , the interactive motion appears to be better than what was previously noted. The line graph in figure 1 (b) demonstrates that at the beginning of the time interval, particles travelled with minimal interaction among themselves; nevertheless, as the time interval extended, they formed groups. At the most recent phases, the interaction improved to  $R = 0.919$ .



**Figure 2:** particles that make up groups  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 20$ ,  $R = 0.996$

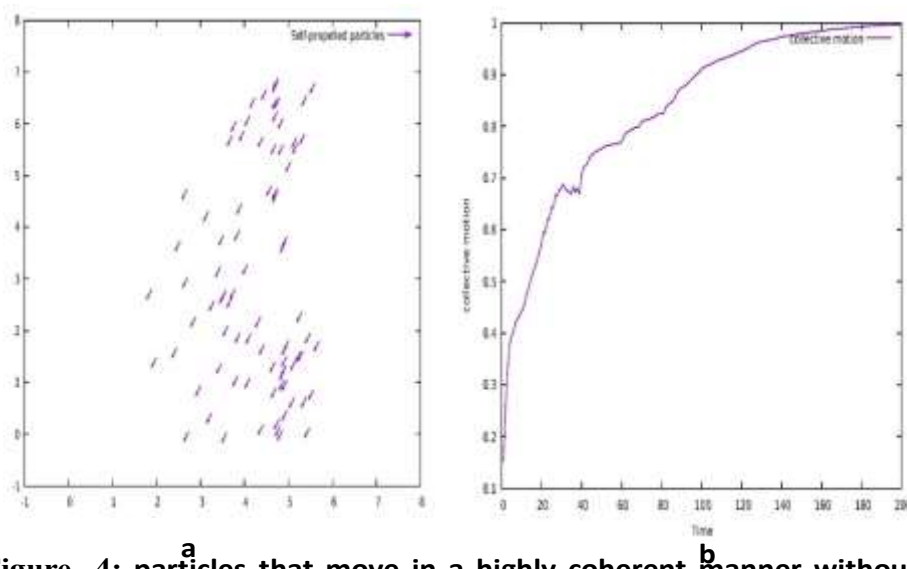
In the aforementioned results, the particles have formed groups when the number of particles is increased from 10 to 20 while maintaining the same other parameters.  $R = 0.996$  indicates an increase in

particle-to-particle contact. Additionally, as velocity in the same direction changed, particle coherence also improved. The line graph in figure 2 (b) demonstrates how particles in their starting state formed highly interacting groups and stayed in those groups until the very end.



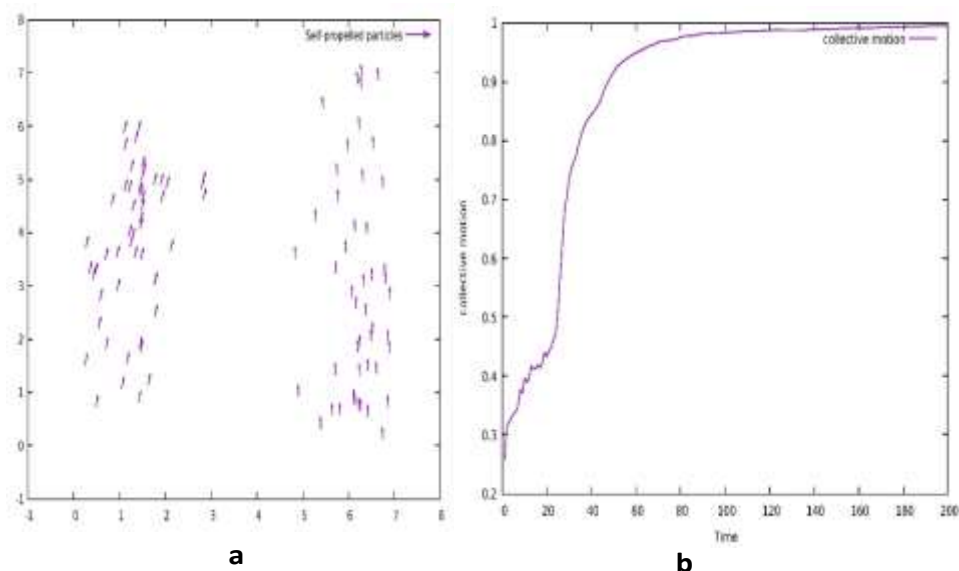
**Figure 3: Interaction and correlation between particles in groups  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 40$ ,  $R = 0.989$**

In this case, the other parameters did not change, but the number of particles grew from 30 to 40. Particles have formed various groups, as seen in figure 3 (a). Although there is a lot of collective motion in groups, the motion's direction varies. Therefore, we might conclude that the particles lack coherence. The graph in figure 3 (b) shows that collective motion in the groups is quite high at the beginning time steps, but as time goes on, the collective motion decreases to individual motion, which explains why short time is crucial for particles to move collectively. When compared to 30 particles, there is less interaction between the particles ( $R = 0.989$ ).



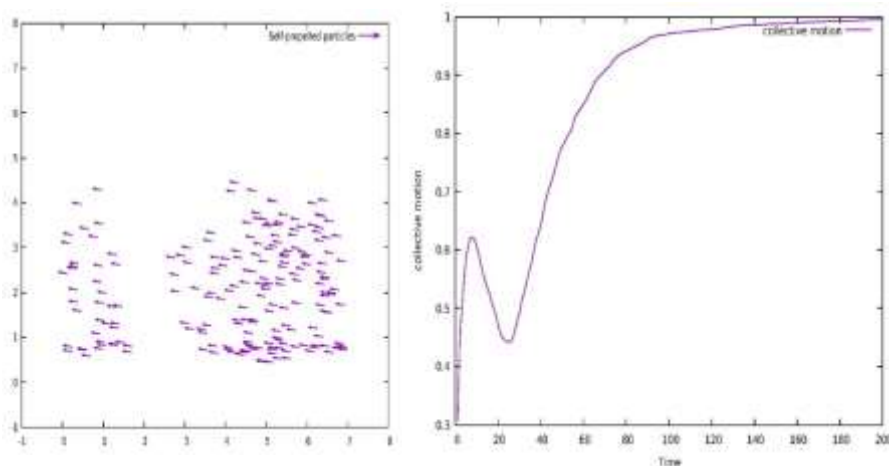
**Figure 4: particles that move in a highly coherent manner without forming any groupings.  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 80$ ,  $R = 0.999$**

Here, quantity 80 is used to measure the particles in figure 4 (a). Although there is little group formation, the particles are travelling with great coherence. It is also evident that there is a lot of interaction between the particles. The particles are travelling in the same direction and not in any other direction. At  $R = 0.999$ , the particle interaction is likewise at its highest point. The line graph in figure 4 (b) shows strong collective motion during the early time intervals. With strong particle contact, the collective motion decreases with increasing time intervals.



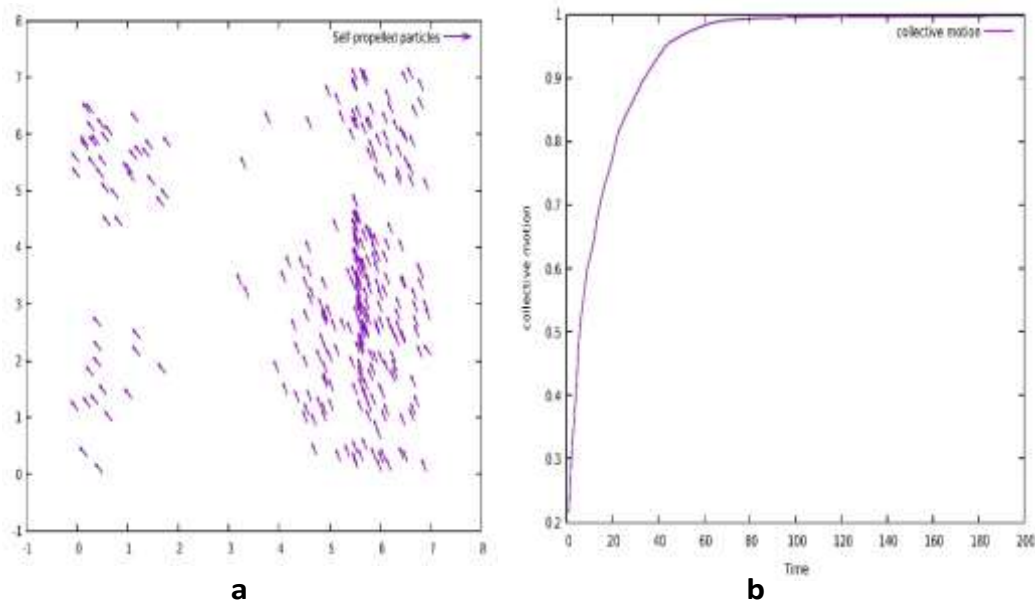
**Figure 5: particles travelling in clusters with correlation  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 100$ ,  $R = 0.999$**

Only the number of particles is increased to 100 in figure 5 (a), but all other parameters remain same. The particles are travelling in groups, and each group is travelling in the same direction. The particles are travelling coherently, but because they are separated into groups, there is less correlation between them. In the groups, there is also a lot of particle interaction. Figure 5 (b) shows that collective motion is lower at first, but as time passes, the interaction relation rises to a high value of  $R = 0.999$ .



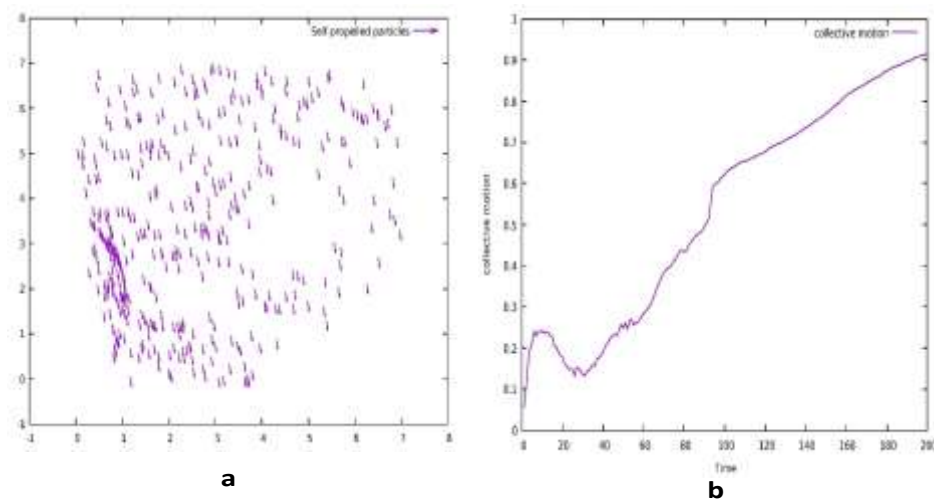
**Figure 6: particles travelling in coherent groups with  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 200$ , and  $R = 0.822$**

Figure 6 (a) uses the number 200 to represent the number of particles. Without much correlation, every particle is travelling in the same direction. This picture illustrates how particles have developed a high degree of coherence among themselves, which is why they are travelling in the same direction even if they are divided into groups. Due to rise in their amount and lower correlation, the interaction between the particles has become less. The collective motion in figure 6 (b) has decreased in comparison to the beginning time as the time interval has increased. Particle interaction also declined, with  $R = 0.822$ .



**Figure 7: Particles exhibiting coherence and forming groups  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 300$ ,  $R = 0.999$**

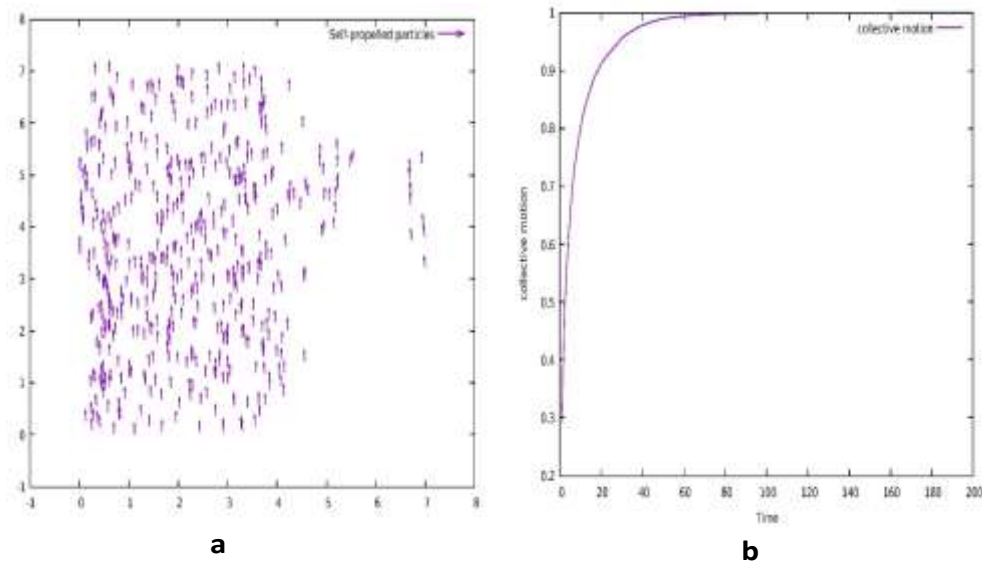
The number of particles is selected up to 300 in figure 7 (a), and other parameters are the same as in earlier experiments. The particles are separated into various groups and move in comparable directions. Although there is a high degree of coherence and collective motion among the particles, the correlation is not as good at that point. There is a lot of contact between the particles in groups. The line graph in figure 7 (b) demonstrates that the particles have been travelling collectively up until this point.



**Figure 8: particles exhibiting a correlation  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 400$ ,  $R = 0.999$**

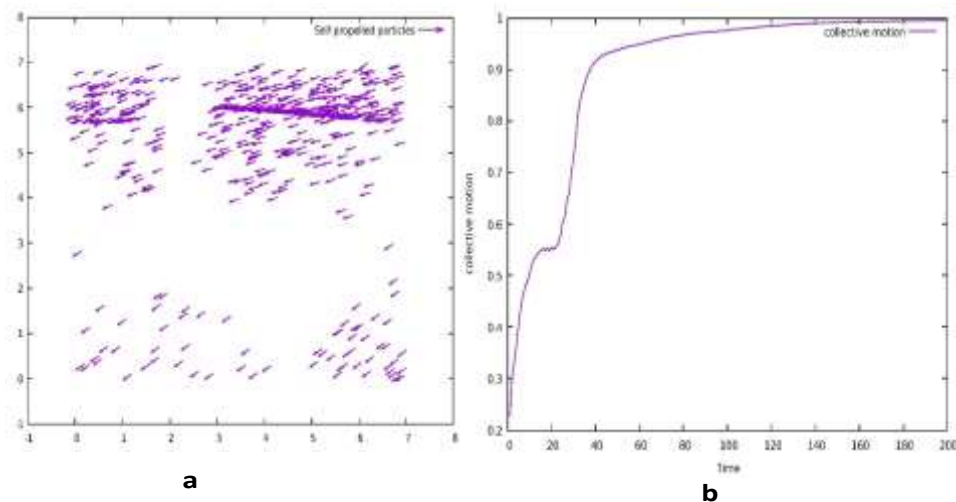


There are up to 400 particles in figure 8 (a). The particles are travelling in unison and in the same direction. The particles are moving in bunches because of their strong connection. They created the groups in order to facilitate the interaction between the particles. Additionally, there is improved particle interaction. The collective motion at the beginning is strong in line graph 8 (b). Particles eventually separated into groups and interval collective motion decreased. Additionally, there is a high level of particle interaction ( $R = 0.999$ ).



**Figure 9: particles exhibiting interaction and coherence  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 500$ ,  $R = 0.997$**

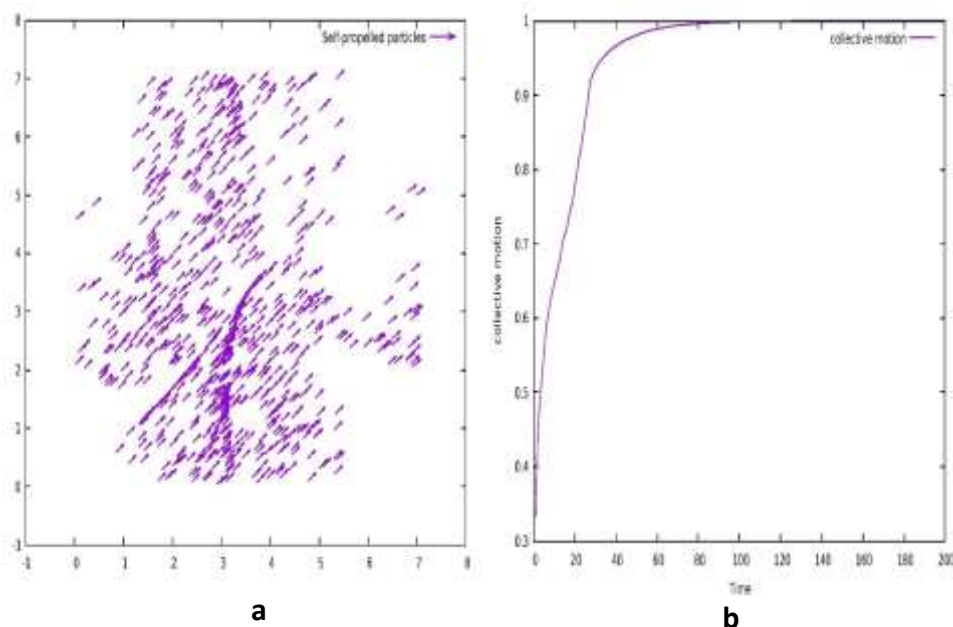
The particles in figure 9 (a) are all moving in the same direction and are numbered with amount 500. The particles are demonstrating a high degree of coherence and have formed several groups. Because of this, there is less interaction between the groups than there used to be. Certain particles are travelling independently and have formed smaller groupings of their own. In the line graph of Figure 9 (b), the particles' initial collective velocity is far higher, but they eventually split into groups. Particle interaction is also decreased.



**Figure 10: particles displaying coherence and interaction  $R = 0.999$ ,  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ , and  $N = 600$**

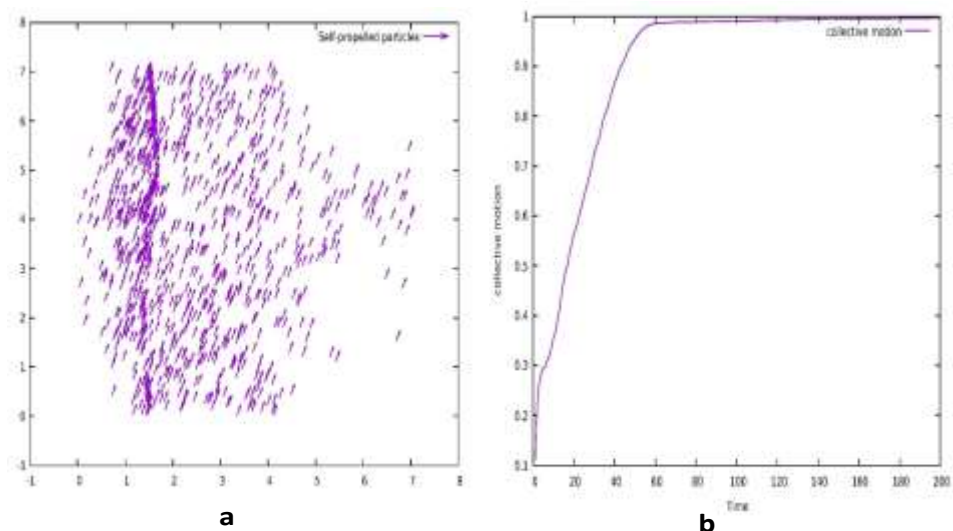


Figure 10 (a) shows 600 particles travelling in groups. While some are migrating independently, they have formed several groups. While there is little correlation between the particles in groups, there is a lot of interaction between them. It is best to move in unison and in the same direction. The line graph in figure 10 (b) illustrates how the collective motion was high at first but decreased as time went on. With a value of  $R = 0.999$ , the particle interaction is extremely high.



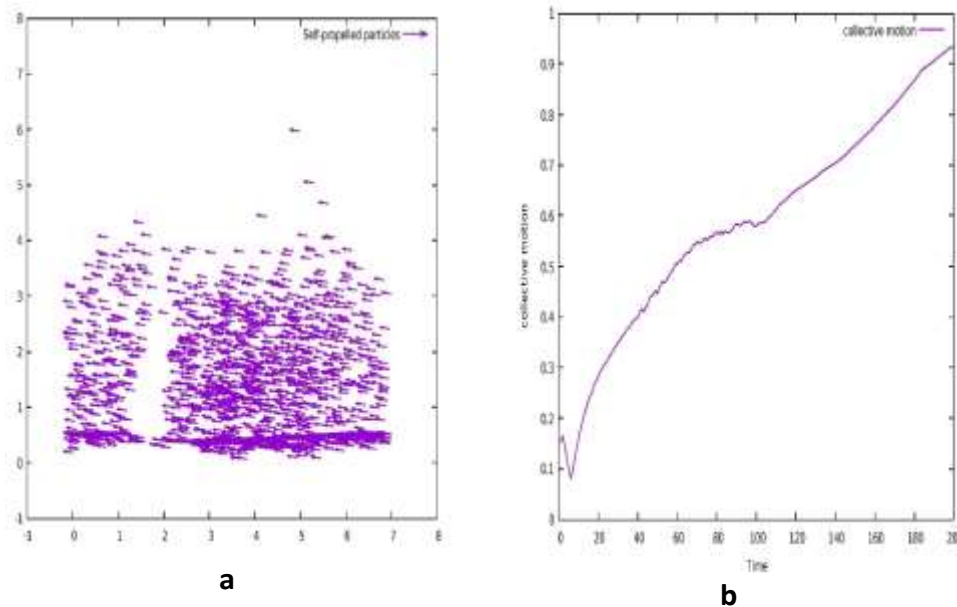
**Figure 11: particles in association with  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 800$ , and  $R = 0.995$**

The number of particles selected in figure 11 (a) is 800. The particles move in the same direction and exhibit excellent coherence. Particles are not separated into larger groupings since there is less space in which they are flowing. There is less interaction between the particles than there was previously. The particles are going in the same direction as a collective. The line graph in figure 11 (b) shows that the particle's collective motion is high at initial time intervals and decreases with time. At this point, the interaction is smaller than in previous collections ( $R = 0.995$ ).



**Figure 12: Particles were grouped according to  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 1000$ , and  $R = 0.999$ .**

A maximum of 1000 particles are taken. The particles have formed numerous groups and have a very powerful collective motion. The particles have a very high link and interaction with one another. When travelling to any other side, the particles only move in one direction. Particle coherence is balanced before decreasing. A small number of particles moved independently without forming groups, demonstrating the correlation gap. The line graph in figure 12 (b) shows how the particles interact and move collectively. The particles are moving collectively at first, but as the time interval lengthens, the effect on collective motion becomes apparent. At this point, there is an extremely strong interaction between the particles ( $R = 0.999$ ).



**Figure 13: particles exhibiting group formation and association  $L = 7$ ,  $\eta = 2$ ,  $t = 200$ ,  $r = 1$ ,  $v = 0.03$ ,  $N = 2000$ ,  $R = 0.999$**

The number of particles is increased to 2000 in figure 13 (a). The particles appeared in correlation and moved into several smaller groupings. The interaction and correlation appear to be more in line with coherence since space is smaller but the particles are more numerous. The particles are travelling in the same direction. Here, we also observe that there is a lot of particle interaction. The particles are collectively moving with time intervals in figure 13 (b). The collective motion decreases as the first time interval begins, but it improves as time increases. At this point, the particle-to-particle interaction is extremely strong, reaching  $R = 0.999$ .

## Conclusion

From the present work it can be concluded that; Low Density ( $N=10$ ) resulted in a disordered state due to occasional particle interactions. Intermediate Density ( $N=20$  to  $100$ ) showed local cluster formation, but global order was not consistently improved, as clusters often moved in divergent directions (non-monotonic behavior). High Density ( $N = 200$ ) consistently achieved a robust, highly ordered phase, with the normalized average velocity ( $v_a$ ) approaching its maximum ( $0.999$ ). This confirms that sufficient density is essential for local alignment to percolate and establish system-wide, coherent motion. This work highlights the essential role of interaction frequency and network connectivity in the self-organization of collective motion.

**Author Contribution**

Murtaza Hussain Shar conceptualized, designed experiments, Abdul Raheem Shar prepared the draft of the article Mazhar Ali Sahito interpreted the data. Dr. Israr Ahmed Memon supervised the whole work. All authors read, revised, and approved the final version of the manuscript.

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**Conflict of Interest**

The authors declare no conflict of interest

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