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ADVANCED NUMERICAL SIMULATION OF MULTILANE TRAFFIC DYNAMICS USING A MODIFIED LWR MODEL WITH RAMPS

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Article Info

Abstract

The traffic flow modeling is a broad area of research that deals with the formulation of mathematical models to understand, predict and design the traffic flow systems. In this research a traffic flow simulation study is conducted by modifying the Lighthill, Whitham and Richards (LWR) model. The main objective is to include the ramps and multi-lanes in the model and analyze their effects on the road sections. The modified model is discretized by using finite difference method (FDM) and applied on a typical road section of length 4KM with four ramps (the road section is adjacent to Quaid-e-Awam University of Engineering, Science and Technology, Nawabshah). Based on the fact that the initial density of cars on road section, with ramps is usually unknown because of instantaneous change therefore a statistical data analysis was conducted. The initial data for a complete week from 7:00 am to 06:00 pm was obtained manually by counting the number of cars entering and leaving from the road sections. From, the data analysis the average values for initial density, velocity and flux were approximated. Then the model is implemented on MATLAB and simulated for the key parameters namely; density of cars, velocity, and flux at different time steps. The simultaneous effect of ramps and multi-lanes of road section were analyzed. The results are useful to predict high density regions on the road section at particular location and at specific time step. The research contributes to predict the behavior of traffic flow phenomenon with multi-lanes and ramps and provides directions for future research. The convergence, well-posed Ness and stability of the numerical schemes have also been analyzed. Finally, the results are validated by comparing with some existing methods and found to be realistic.

Keywords:

Traffic Dynamic, Simulation, Multilane, Macroscopic, Ramps.

INTRODUCTION

The traffic flow theory involves the development of mathematical models among the primary elements of a traffic stream: Flow density ρ (vehicles/km), speed v (km/hr) and flux Q (vehicles/hr). These aspects of traffic flow modeling can be used in the planning, design, and operation of highway system [1]

Traffic Flow modeling is a tool which is used to understand and express the properties of traffic flow. While the millions of vehicles/cars are on the roadways/highways. These vehicles/cars work together with each other and effect the overall flow/movement of traffic, or the traffic flow [2]

From a mathematical perspective, modeling of physical phenomena in a deterministic sense is crucial for predicting future states of models/systems. The flow mechanism of vehicles is associated with shocks, density and velocities which can be modeled using both ordinary differential equation (ODE's) and partial differential equations (PDE's). For the efficiency of the current traffic flow models, the mathematics establish the new traffic models and determine whether the model has difficulty to providing practical traffic flow in the real world. The traffic flow modeling is first described by the US-American Bruce D. Greensheild after the practical observation at the Yale Bureau of highway traffic in 1930s [3] First time he carried out the relation between the traffic flow, density and speed using photographic method and defined as; Flow= density*speed i.e., $Q = \rho v$. Since then the traffic flow models became more popular and became very important practically. Moreover, in 1950s the Glen Wardrop [3] an English transport analyst gave the theory of traffic modeling during the development of the first and second principles of equilibrium. A bulk traffic flow is shown in figure 1

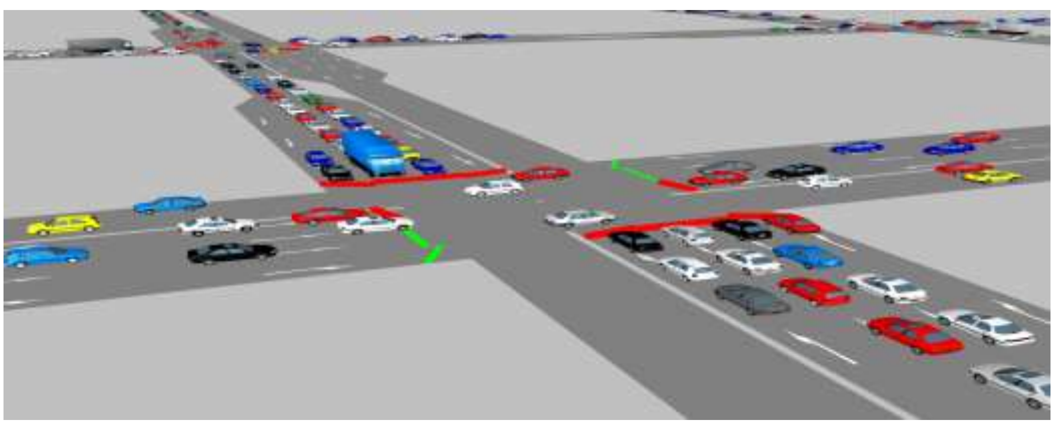

Figure 1. Bulk Traffic Flow

Traffic flow modeling on multilane freeways with ramps is a critical problem due to complex interactions among lane-changing, merging, congestion propagation, and capacity reduction at bottlenecks. Bulk-flow (macroscopic) models have been widely used to analyze such systems because of their computational efficiency and theoretical soundness [4]. However, most traditional models such as Lighthill–Whitham–Richards (LWR) and the Cell Transmission Model (CTM) primarily address single-lane dynamics and do not fully capture multilane interactions and ramp-induced phenomena [5,6]. Early efforts to incorporate lane-changing in macroscopic models were made by Holden and Risebro, who developed a weakly coupled hyperbolic formulation suitable for an arbitrary number of lanes [7]. Daganzo extended this work by proposing analytical merge and lane-change source terms to represent the impact of on-ramps [8, 9]. Empirical studies over the lane changes at merge bottlenecks affect capacity and flow dynamics done by [10-12]. Capacity drop due to merging has been deeply analyzed using extensions of CTM. For instance, [13] introduced a ramp-aware macroscopic model that captures stop-and-go waves and instability in multi-ramp highway sections. [14] introduced the analytical expressions for capacity drop at multilane merge points. Empirical observations given by [15] and [16] substantiate the theoretical understanding of capacity loss in congested merge zones. Control strategies aiming to mitigate merge congestion include a variety of traffic management techniques. Further studied integrated ramp metering and variable speed limit control, showing notable improvements in mainline stability. [17] studied the stochastic numerical simulation of traffic flow which does not incorporate the multilane and ramps. Ramp metering, especially the ALINEA algorithm and its extensions, have been effective in reducing congestion and emissions [18-20]. A recent comprehensive survey established that adaptive metering combined with Connected and Autonomous Vehicle (CAV) technologies promises further performance gains [21]. CAV-based coordination at on-ramps has gained traction in traffic research. Further formulated a flow-level coordination framework for multilane merging scenarios using CAVs, resulting in improved stability and throughput. Complementing this, [22] reviewed various CAV merging strategies, distinguishing between single-lane and multilane freeway contexts. On the microscopic side, [23] presented a data-driven calibration of on-ramp merging in a hybrid macroscopic context. Numerous microscopic and mesoscopic studies have also informed macroscopic modeling. [24] studies the macroscopic changes due to ramp turbulence and weaving flows. Additional microscopic reviews on driver behavior during merging provide essential empirical inputs [25]. [26] introduced multilane-specific macroscopic models that consider lane width and count, enriching the model's realism. Despite significant advancements, a unified framework that seamlessly integrates multilane bulk-flow dynamics**,** ramp-induced capacity drop**,** lane-changing behavior**,** and optional CAV coordination has not yet been established. Existing macroscopic models often ignore critical ramp behavior; microscopic models, while detailed, lack scalability; CAV control solutions remain siloed [27, 28]. The main aim of this paper is to consider the macroscopic classical LWR model and include the effects of multi-lane and ramps, discretize the formulated model using finite difference method and apply on a physical multi-lane road with ramps and then predict the high density and high velocity regions on multi-lane and ramps by solving the proposed model. This study considers the flow of cars on highways with multi-lane and ramps, but all other kinds of vehicles are ignored. Also the condition of roads, signals is not incorporated in the model. Cars are considered as fluid particles and their free motion is simulated as a bulk of fluid particles.

Mathematical Model:

The continuity equation describes the conservation of vehicles in terms of the traffic density and flow density relation. Consider the $\rho(x,t)$ that denotes the density of cars at any location x and at specific time t. The single-lane densities are $\rho_i(x,t)$ on lane I for $n = 1, 2, 3, ..., n$ and the total density over all lanes is denoted by $\rho_{tot}(x,t)$. Then the lane-average density/effective density is

$$\rho(x,t) = \frac{\rho_{tot}(x,t)}{I}, \qquad \textbf{(1)}$$

$$\rho_{tot}(x,t)=\sum_{i=1}^{I}\rho_i(x,t)=I\rho(x,t), \tag{2}$$

and

$$Q_i(x,t)=\rho_i(x,t)\,\mathrm{V}(x,t) \tag{3}$$

Where Q and V denote the flux and velocity respectively. Thus the average & sums for flux over all lanes is given by as follows:

$$Q_{tot}(x,t)=\frac{1}{I}\sum_{i=1}^{I}Q_i(x,t)=\frac{Q_{tot}}{I}, \tag{4}$$

Finally, the flow relation can be defined as

$$Q_{tot}(x,t)=\rho_{tot}(x,t)\,\mathrm{V}(x,t), \tag{5}$$

In addition, if the section of road is homogeneous so the traffic flow will be at the boundary then flows at x and $x+\Delta x$ and $Q_{tot}(x,t)$ and $Q(x+\Delta x,t)$ respectively. Then change in the number of cars $n(\mathrm{t})$ is given by,

$$\frac{dn(\mathrm{t})}{dt}=Q_{in}(\mathrm{t})-\mathrm{Q}_{\mathrm{out}}(\mathrm{t})=Q_{tot}(x,t)-Q(x+\Delta x,t), \tag{6}$$

which simplifies to,

$$\begin{aligned}\frac{\partial\rho_{tot}(\mathrm{x},\mathrm{t})}{\partial t}=\frac{1}{\Delta x}\frac{dn}{dt}&=-\frac{Q_{tot}(x+\Delta x,t)-Q_{tot}(x,t)}{\Delta x}\\&=-\frac{\partial Q_{tot}(\mathrm{x},\mathrm{t})}{\partial x}\end{aligned} \tag{7}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho V)}{\partial x}=0. \tag{8}$$

Equation (8) is known as the continuity equation for homogeneous road section which describes the non-steady flow of cars.

The continuity equation with the on and off ramps can be formulated from Eq. (8) as follows:

$$\frac{\partial\rho_{tot}}{\partial t}+\frac{\partial(\rho_{tot}V)}{\partial x}=\frac{Q_{ramp}}{L_{ramp}}, \tag{9}$$

Where L_{ramp} is length between inflow and outflow and Q_{ramp} is the flow from ramps, thus,

$$\frac{d}{dx}Q_{ramp} = \frac{Q_{ramp}}{L_{ramp}}, \tag{10}$$

So the continuity equation with ramps becomes as follows;

$$\frac{\partial \rho_{tot}}{\partial t} + \frac{\partial(\rho_{tot}V)}{\partial x} = \frac{Q_{ramp}}{L_{ramp}} = I\upsilon_{ramp}(\mathrm{x},\mathrm{t}), \tag{11}$$

where, $\upsilon_{ramp}(\mathrm{x},\mathrm{t}) = \begin{cases} \dfrac{Q_{ramp}(\mathrm{x},\mathrm{t})}{IL_{ramp}}, & \text{if x is within merging or diverging zone} \\ 0 & \text{, otherwise} \end{cases}$,

by dividing the number of lanes Eq. (11) reduces to;

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho V)}{\partial x} = \upsilon_{ramp}(\mathrm{x},\mathrm{t}), \tag{12}$$

which is the continuity equation for the ramps.

The continuity equation for change in each lane would be coupled by source terms along with the lanes I (x), where I (x) is change in lanes over length**.** Using the relations for flux and density for single lane $Q(\mathrm{x},\mathrm{t}) = \dfrac{Q_{tot}(x,t)}{I(\mathrm{x})}$ and $\rho(\mathrm{x},\mathrm{t}) = \dfrac{\rho_{tot}(x,t)}{I(\mathrm{x})}$, respectively, if we consider the two lanes with densities $\rho_1(x,t)$ and $\rho_2(x,t)$ then the LWR model is modified and yields the following system of PDEs,

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} + \frac{\partial Q_1}{\partial x} = 0 \\ \frac{\partial \rho_2}{\partial t} + \frac{\partial Q_2}{\partial x} = 0 \end{aligned}, \tag{13}$$

where $Q_1(x,t)$ and $Q_2(x,t)$ are the respective fluxes for each lane. Now considering the velocity of vehicle $\upsilon(\mathrm{x},\mathrm{t}) = \beta(\rho_{\max} - \rho)$. Further, by considering the driver's decision, the velocity of vehicle is given by the following Eq. (14) as

$$\upsilon(\mathrm{x},\mathrm{t}) = \beta(\rho_{\max} - \rho) - \alpha\left(\frac{\partial \rho}{\partial x}\right), \tag{14}$$

where α and β are the constants. Thus the conservation equation will become as follows,

$$\frac{\partial \rho}{\partial t} + V(\rho)\frac{\partial \rho}{\partial x} = \alpha\frac{1}{2}\frac{\partial^2 \rho^2}{\partial x^2}, \tag{15}$$

Consequently, for the two lanes model without any interaction, the model equations are obtained as below,

$$\frac{\partial \rho_1}{\partial t} + V(\rho_1)\frac{\partial \rho_1}{\partial x} = \alpha \frac{1}{2}\frac{\partial^2 \rho_1^{\,2}}{\partial x^2}, \quad \textbf{(16)}$$

$$\frac{\partial \rho_2}{\partial t} + V(\rho_2)\frac{\partial \rho_2}{\partial x} = \alpha \frac{1}{2}\frac{\partial^2 \rho_2^{\,2}}{\partial x^2}, \quad \textbf{(17)}$$

If $E(x,t) = \mu(\rho_2 - \rho_1)$ represent the change in the lane then,

$$\begin{aligned} \frac{\partial \rho_1}{\partial t} + \frac{\partial Q_1}{\partial x} &= E(x,t) \\ \frac{\partial \rho_2}{\partial t} + \frac{\partial Q_2}{\partial x} &= -E(x,t) \end{aligned}, \quad \textbf{(18)}$$

Hence the modeled equations that govern the traffic flow are (8), (12), (16), (17) and (18). The initial and boundary conditions for these equations require the values of initial car density at the road section and the velocity of cars at the inlet and outlet boundaries.

In order to solve the traffic flow equations numerically the model equations (8), (12), (16), (17) and (18) are discretized by the explicit upwind finite difference method which is 2nd order accurate. The discretized form of the governing equations is listed below:

$$\frac{\rho(i,j+1)-\rho(i,j)}{\Delta t} + \frac{V(i,j)(\rho(i+1,j)-\rho(i,j))}{\Delta x} + \frac{\rho(i,j)(V(i+1,j)-V(i,j))}{\Delta x} = 0, \quad \textbf{(19)}$$

$$\frac{\rho(i,j+1)-\rho(i,j)}{\Delta t} + \frac{V(i,j)(\rho(i+1,j)-\rho(i,j))}{\Delta x} + \frac{\rho(i,j)(V(i+1,j)-V(i,j))}{\Delta x} = \upsilon_{ramp}, \quad \textbf{(20)}$$

$$\frac{\rho_1(i,j+1)-\rho_1(i,j)}{\Delta t} + V(i,j)\left(\frac{\rho_1(i+1,j)-\rho_1(i,j)}{\Delta x}\right) = \frac{1}{2}\frac{\alpha\left(\rho_1(i+1,j)-2\rho_1(i,j)+\rho_1(i-1,j)\right)}{\Delta x^2}, \quad \textbf{(21)}$$

$$\frac{\rho_2(i,j+1)-\rho_2(i,j)}{\Delta t} + V(i,j)(\frac{\rho_2(i+1,j)-\rho_2(i,j)}{\Delta x}) = \frac{1}{2}\frac{\alpha(\rho_2(i+1,j)-2\rho_2(i,j)+\rho_2(i-1,j)}{\Delta x^2}, \quad \textbf{(22)}$$

$$\frac{\rho_1(i,j+1)-\rho_1(i,j)}{\Delta t} + V(i,j)\left(\frac{\rho_1(i+1,j)-\rho_1(i,j)}{\Delta x}\right) + \rho_1(i,j)(\frac{V(i+1,j)-V(i,j)}{\Delta x}) = \mu(\rho_2(i,j)-\rho_1(i,j)), \quad \textbf{(23)}$$

$$\frac{\rho_2(i,j+1)-\rho_2(i,j)}{\Delta t} + V(i,j)\left(\frac{\rho_2(i+1,j)-\rho_2(i,j)}{\Delta x}\right) + \rho_2(i,j)(\frac{V(i+1,j)-V(i,j)}{\Delta x}) = \mu(\rho_2(i,j)-\rho_1(i,j)). \quad \textbf{(24)}$$

Well-posed Ness and Stability Condition.

This section discusses the well-posed Ness and stability conditions for the traffic flow. Consider the flux is defined as $Q = \rho v$, where ρ is the density and v is the speed/velocity of vehicles, and both are the functions of time t and space x. The velocity for modeled equation (14) is defined as $v(\rho) = \beta(\rho_{\max} - \rho)$ and then the flux became as,

$$Q = \rho v(\rho) = \rho\{\beta(\rho_{\max} - \rho)\} \quad , \tag{25}$$

or

$$Q = \beta(\rho\,\rho_{\max} - \rho^2) \ , \tag{26}$$

Now by using upwind explicit finite difference method it is assumed that the cars are moving in one direction for the characteristic speed $\frac{dv}{dt}$ that must be positive.

$$Q'(\rho_{i,j+1}) = \beta(\ \rho_{\max} - 2\rho_{i,j+1}) \geq 0 \ , \tag{27}$$

after simplification the following equation is obtained.

$$\rho_{\max} = 2\rho_{i,j+1} . \tag{28}$$

Equation (28) represents the well-posed Ness of the traffic flow model. The proof of the following proposition further supports the well-posed Ness and stability criteria.

Preposition:

The well-posed Ness and stability condition for explicit upwind difference scheme is given by the following two conditions respectively,

$$\rho_{\max} = c \max_k \rho_o(x_k),\ c = 2 \quad \text{and } \lambda = \rho_{\max} \frac{\Delta t^{n+\frac{1}{2}}}{\Delta x_i} \leq 1 \ , \tag{29}$$

Proof:

Referring to the conservation equation (8) for the traffic flow, that is

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial x} = 0 \ , \tag{8}$$

or,

$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0 \qquad \therefore \mathrm{Q}(\rho) = q(\rho) = \rho v \ , \tag{30}$$

where,

$$Q(\rho) = \beta(\rho\,\rho_{\max} - \rho^2), \tag{31}$$

then Eq. (8) can be also be writing as follows,

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0 , \tag{32}$$

Then finite difference scheme for Eq. (32) is as follows,

$$\rho_i^{n+1} = \rho_i^n - q'(\rho_i^n)\frac{\Delta t^{n+\frac{1}{2}}}{\Delta x_i}[\rho_i^n - \rho_{i-1}^n] \quad , \tag{33}$$

where, $$q_i^n = \rho_i^n \beta(\rho_{\max} - \rho_i^n), \tag{34}$$

then Eq. (34) leads to,

$$\Rightarrow \rho_i^{n+1} = (1-\lambda)\rho_i^n + \lambda\rho_{i-1}^n \quad , \tag{35}$$

where, $$\lambda := \mathrm{q}'(\rho)\frac{\Delta t^{n+\frac{1}{2}}}{\Delta x_i}, \tag{36}$$

Now if $\lambda \le 1$, then the new solution will be the combination of two previous solutions, which is at time step $n+1$ at node i, is an average of the solutions at the previous time step at node i and $i+1$. This means that the extreme values of previous two solutions lie at two consecutive nodes. Therefore, the new solution continuously dependent on the initial value $\rho_i^0 \ \ i = 1, 2, 3, \ ..., \ M$ and the explicit upwind finite difference scheme is stable justified by the relation,

$$\lambda := \mathrm{q}'(\rho)\frac{\Delta t^{n+\frac{1}{2}}}{\Delta x_i} \le 1 \quad , \tag{37}$$

or $$\lambda := \rho_{\max}\frac{\Delta t^{n+\frac{1}{2}}}{\Delta x_i} \le 1 \quad , \tag{38}$$

This is the stability condition for the numerical solution of the model.

Methodology:

Computational Domain and Problem Space.

To apply the model on a physical road section a computational domain consisting of a typical road section of length 4 Kilometer passing through QUEST, Nawabshah is selected. There are total of four pairs of ramps (ramps on and ramps off). The upstream flow of cars starts from Ali Shah (inlet position) and ends at the New Naka (outlet position) on up lane denoted by L_u with two multilane L_{ul} (left lane up) and L_{ur} (right lane up). The on and off ramps on L_u are also specified at inlet and outlet. Similarly, the downstream flow of cars starts from New Naka (inlet position) and ends at Ali Shah (outlet position) on down lane denoted by L_d with two multilane L_{dl} (left lane down) and L_{dr} (right lane down). The on and off ramps on L_d are also specified at inlet and outlet. The numerical simulation of the problem requires the prior information interms of the constant parameters, initial density on the road which is quite difficult to know unless there is instantaneos data from the satelite sensor is provided. Also, due to the random nature of the flow of cars on roads the exact initial density is not know but it can be approximated by manual data collection on the road section particularly on inlet, outlet and ramps.

Data Collection and Statistical Analysis.

In order to obtain the realistic traffic simulation results the initial data in terms of average car density is required. In most of the cases the data is collected directly by GPS or satellite sensors. But in the case of unavailability of such devices the manual data collection procedure can be used. To obtain an approximate value on the selected road geometry, the number of cars entering and leaving on the up/down lanes and on/off ramps is recorded on a typical week. Then the statistical analysis is done for each road section which will be useful as initial condition in the implementation of model. The data is collected manually by recording the inflow and out flow of cars at different road sections. For a typical week of June 2025 the data is recorded from 7:00 to 18:00 hr and then the statistics of data is calculated according to hour-wise, day-wise and week-wise. The table 1 shows the overall analysis on the section of road.

Road Section	Average	Range	Standard Deviation	Skewness
UP (In)	32.89	11.38	34.38	0.09
Up (out)	71.73	18.94	57.81	0.35
Down (In)	47.87	18.05	57.00	0.34
Down (out)	31.39	12.05	37.99	0.29
R1u_on	14.28	7.11	20.87	0.44
R1u_off	14.92	5.65	16.52	0.13
R2u_on	24.64	5.56	17.08	0.11
R2u_off	17.85	5.78	18.29	0.48
R1d_on	10.05	3.12	9.78	0.00
R1d_off	9.58	3.05	9.71	0.09
R2d_on	2.11	1.45	4.17	0.52
R2d_off	1.69	1.29	3.63	0.35

Table 1: Overall Analysis of the Road Number of Cars on Road Section

After the detailed data analysis the overall analysis of data is visualized in the following Figure 2. It can be seen that the highest average number of cars, standard deviation and range is found on the Up_Lane (outlet) section of roads because of the more contribution from the R2u_on and R1d_on. Then Down_Lane (inlet), Down_Lane (oulet) and Up_Lane (intlet) are contributing to the traffic flow density. From the ramps R2u_on is contributing more than other ramps while the R2d_on and R2d_off have less number of cars recorded due to not much population on that link road side. The trend lines are the approximated fits through the collected data that show the increasing/decreasing behavior of cars on different road sections

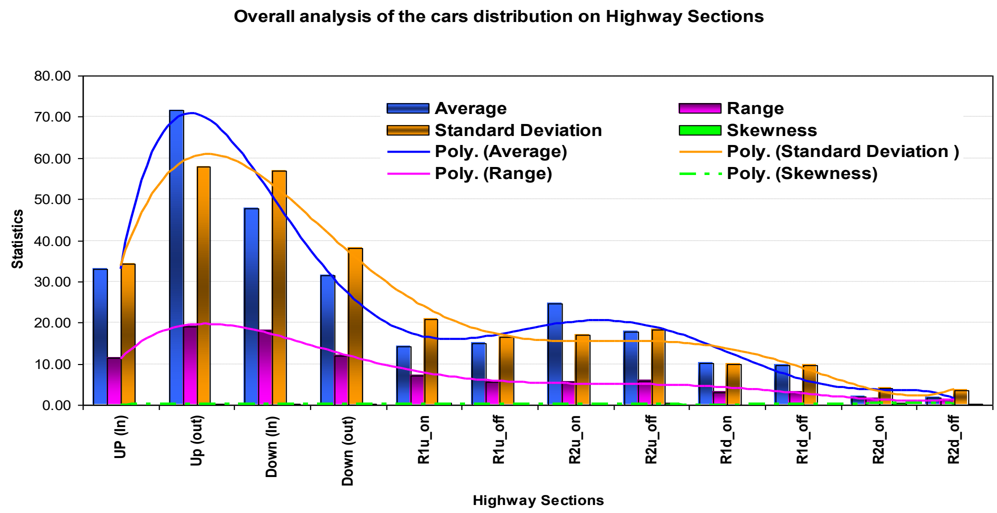

Figure 2: Overall Cars Distribution on Different Road Sections Per Week

Initial Results from Statistical Analysis

Since the main objective of the statistical data analysis is to find the average initial density of cars on the UP_Lane and Down_Lane so that the realistic simulation results be obtained. For this purpose, the 4KM lane is distributed into 40 segments each segment having 0.1 KM and the maximum number of cars at any initial time t=0 are distributed at each segment. The approximated initial density for Up_Lane and Down_Lane is presented in Figure 3 and Figure 4 respectively. From figure 3 it can be seen that the density of cars on Up_Lane fluctuates between 3 and 5 per 0.1 KM. While on Down_Lane the density of cars remains within 4 cars per 0.1 km.

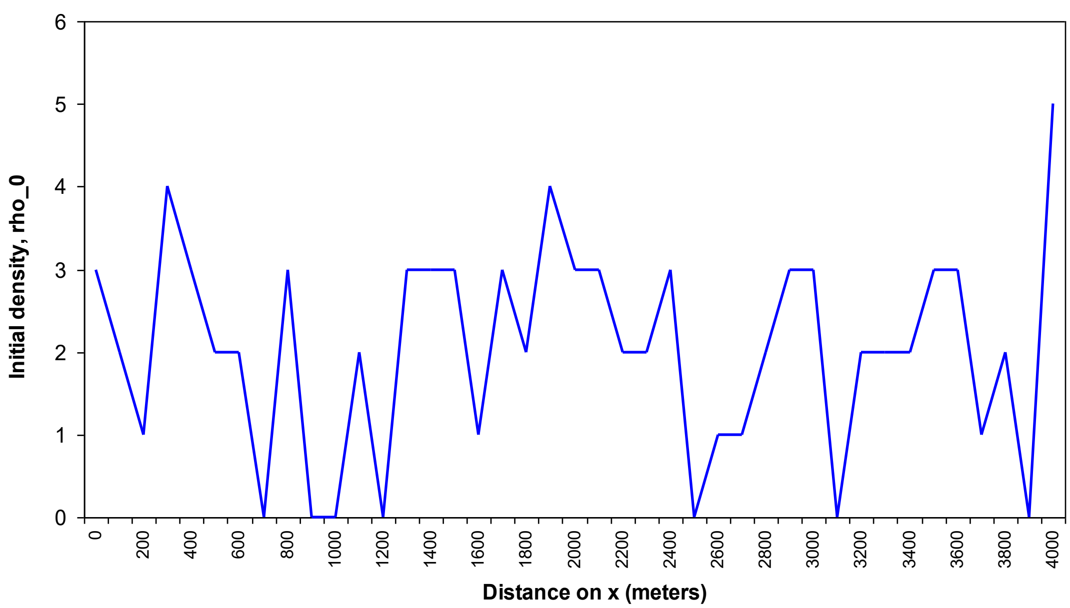

Figure 3: Average Initial Density of Cars at 4km UP_Lane.

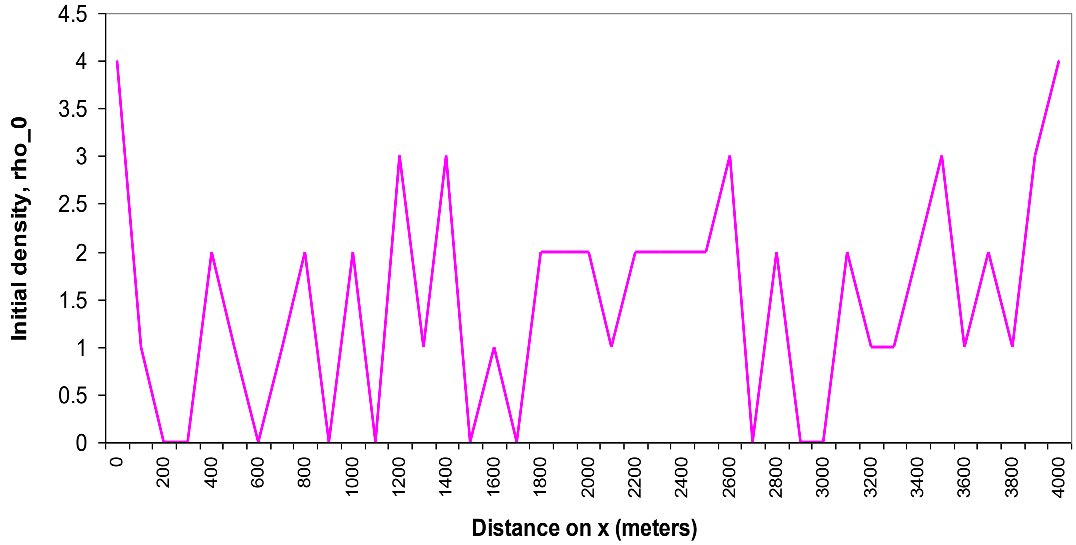

Figure 4: Average Initial Density of Cars at 4km Down_Lane

Results and Discussion

This section devoted to the numerical simulation of the formulated traffic flow model. The governing set of discretized equations (3.19-3.24) are solved by writing a user defined code in MATLAB. The simulation results are obtained for different time steps. The results predict the density of cars on Up_Lane, Down_Lane and both left and right lanes. The simulation results are also validated by comparing the results of [30]. In addition the convergence and stability of the output parameters also have been discussed.

Testing and Implementation.

For the testing and implementation purpose the following data is used in the model to obtain the local traffic simulation on Up_Lane.

Length of road section (UP), $x = 4\ km,\ \Delta x = 100,\ n = 40,\ \Delta t = 1$
Inlet boundary $\rho(0,t) = 3\, cars / 0.1\, km,\ \alpha = 1,\ \beta = 1.$
$\rho_{\max} = \rho(x,0).c = 20\, cars / km,\ c = 1.$
At inlet $V_{\max} = 60\ km / hour,$ At outlet $V_{\max} = 30\ km / hour,\ error\, tol = 0.0001$

and for the Down_Lane the following initial data is used.

Length of road section (Down), $x = 4\ km,\ \Delta x = 100,\ n = 40,\ \Delta t = 1$
Inlet boundary $\rho(0,t) = 4\ cars / 0.1\ km,$
$\rho_{\max} = \rho(x,0).c = 24\, cars / km,\ c = 1.5.$
At inlet $V_{\max} = 20\ km / hour,$ At outlet $V_{\max} = 55\ km / hour,\ error\ tol = 0.0001$

After the complete formulation of the model and all necessary information used the simulation is run for 5 minutes or 300 sec for Up_Lane and the local density profiles are shown in Figure 5. Which shows the variation in the density of cars at different locations of road section the red color regions indicate the maximum density and the blue color regions indicate the minimum density. And the density value fluctuates between 0 and 3. Similarly, the simulation is run for 5 minutes or 300 sec for Down_Lane and the local density profiles are shown in Figure 6, shows the variation in the density of cars at different locations of road section where the density values fluctuate between 0 and 3. The effect of multilane for each upstream and downstream lane is also simulated individually. The distribution of density of cars on both up_left lane and up_right lane is exhibited in figure 7 and it can be compared that the right lane has more density than the left lane and the values fall between 0 and 3. In a similar way the distribution of density of cars on both down_left lane and down_right lane is exhibited in Figure 8 and it can be compared that the right lane has more density than the left lane whose values fall between 0 and 2.5.

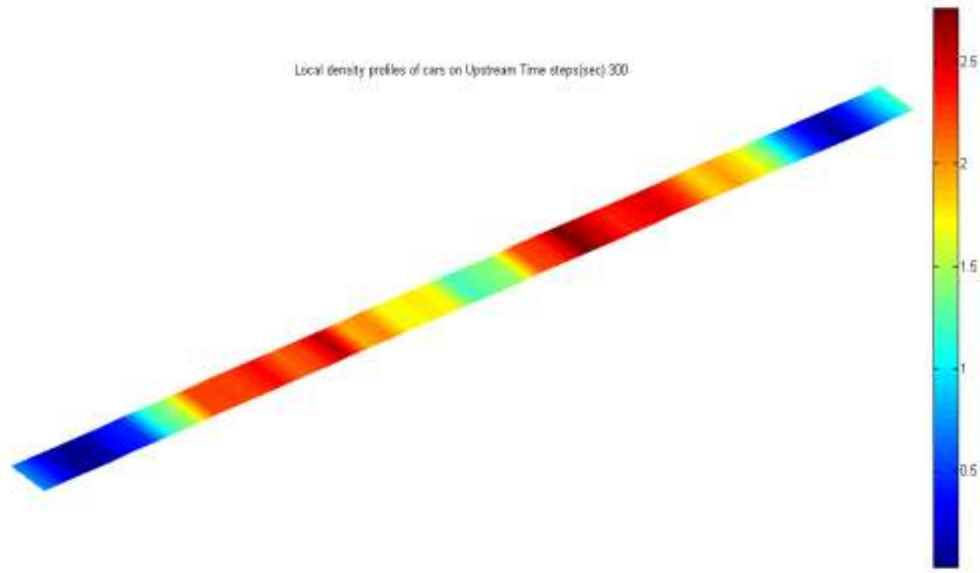

Figure 5: The Local Density Profiles of Cars on Up_Lane After 300 Time Steps

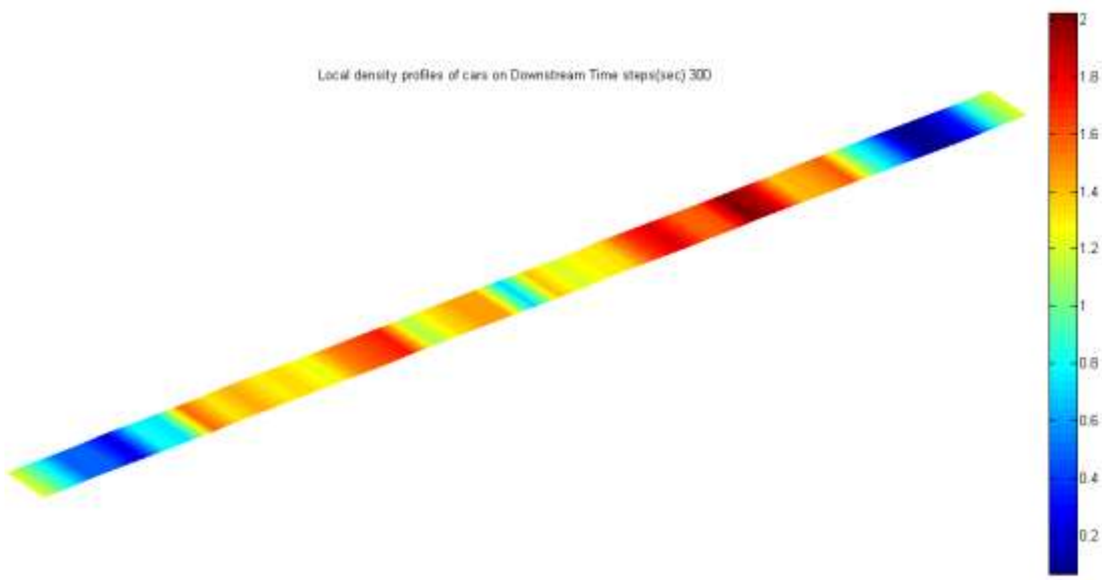

Figure 6: The Local Density Profiles of Cars on Down_Lane After 300 Time Steps

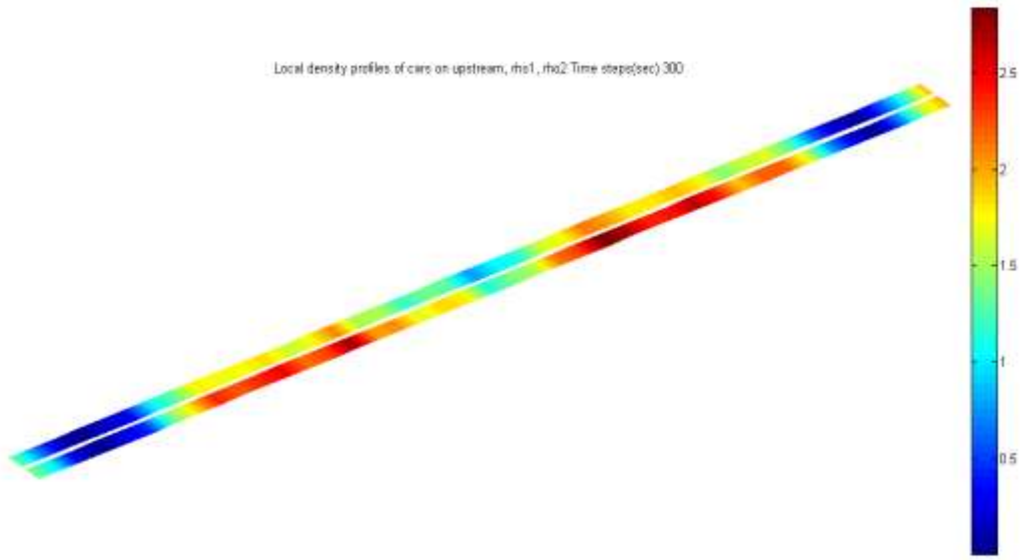

Figure 7: The Local Density Profiles of Cars on Up_Lane (Up_Left and Up_Right Lane) After 300 Time Steps

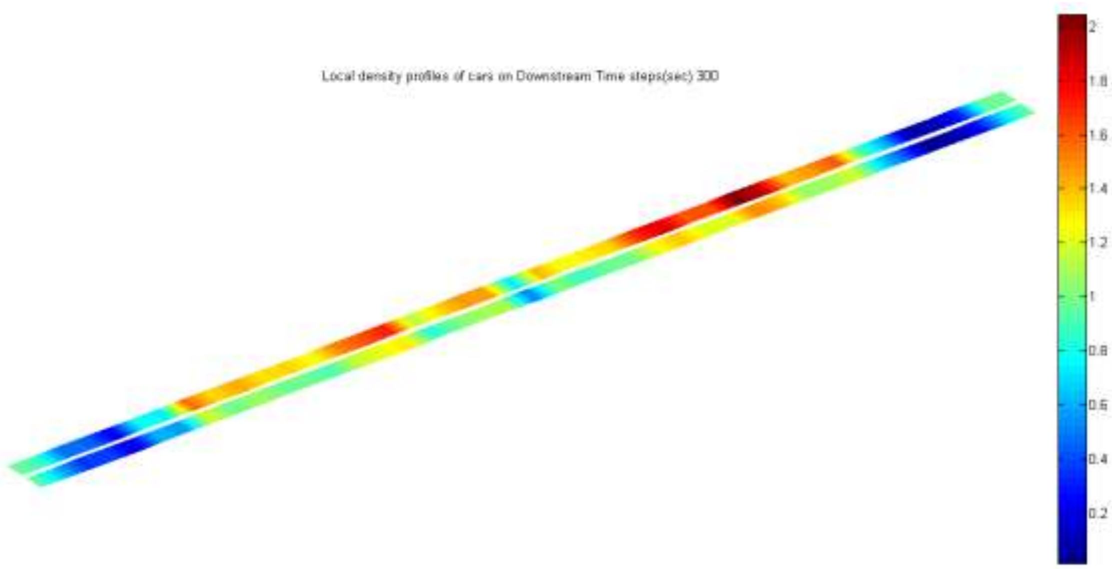

Figure 8: The Local Density Profiles of Cars on Down_Lane (Down_Left and Down_Right Lane) After 300 Time Steps

In order to obtain the simultaneous effect of density (rho) of ramps and multilane on the Up_Lane is simulated and presented in the following Figure 9. It can be observed that R1u_on is contributing more than other road sections and second highest effect is from R2u_off. While at outlet R2u_on has the maximum value.

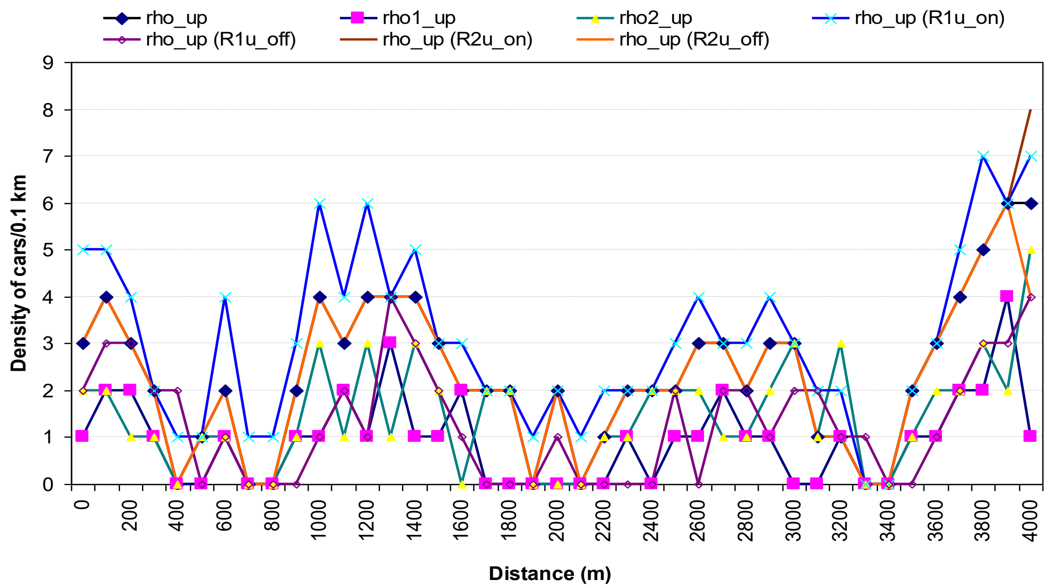

Figure 9: The Density Effect of Ramps and Multilane on UP_Lane After 300 Time Steps

The simultaneous effect of velocity (V) of ramps and multilane on the Up_Lane is simulated and presented in the following Figure 10. It can be seen that the velocity up_left lane remains maximum at both inlet and outlet while R1u_on has minimum velocity at inlet and R2u_on has minimum velocity at outlet.

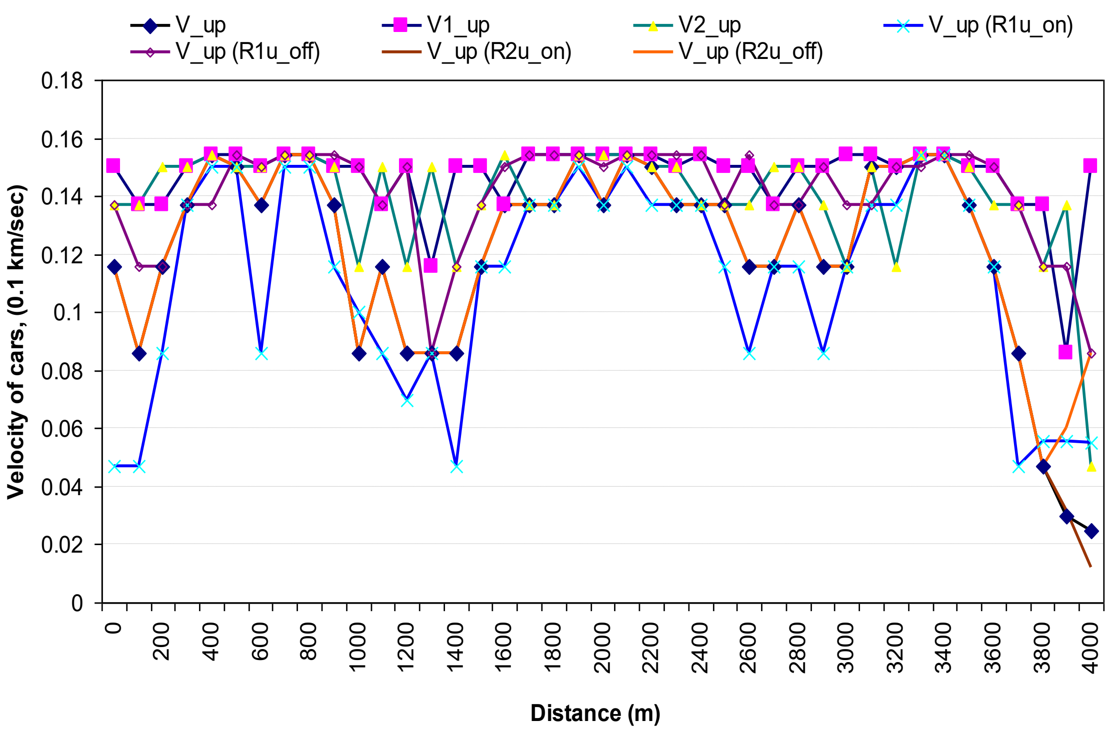

Figure 10: The Velocity Effect of Ramps and Multilane on UP_Lane After 300 Time Steps

Similarly, the simultaneous effect of flux of cars (Q) of ramps and multilane on the Up_Lane is simulated and presented in the following Figure 11. The maximum flux can be seen on R1u_on at 1000m, it is because of the high initial density at this point. Furthermore, R2u_off has maximum flux at inlet and R1u_on has maximum flux at outlet.

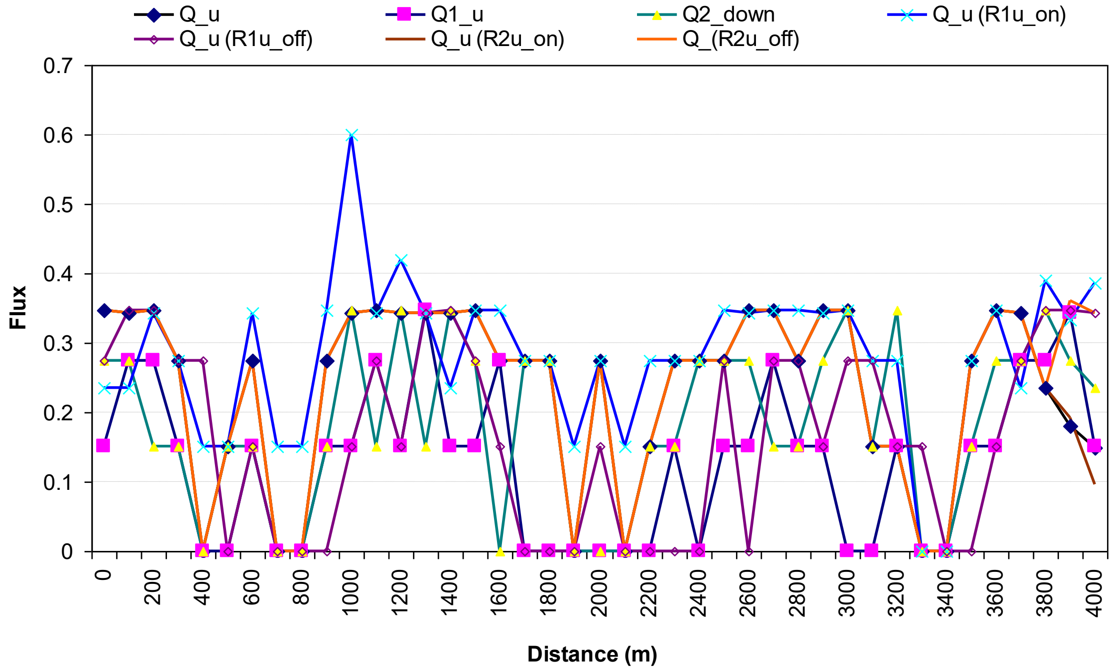

Figure 11: The Flux Effect of Ramps and Multilane on UP_Lane After 300 Time Steps

Validation of Results.

The results are compared with the work of [23] who gave the relation $q(\rho)=(\rho V_{\max}(1-(\frac{\rho}{\rho_{\max}})^2))$ for finding the flow of cars/vehicles. The comparison is shown by Figure 12 and results are found to be realistic with maximum absolute error 0.15 which is acceptable. Fluctuation in the results is the cause of the nonlinearity in the relation and the different methods used to solve the problem.

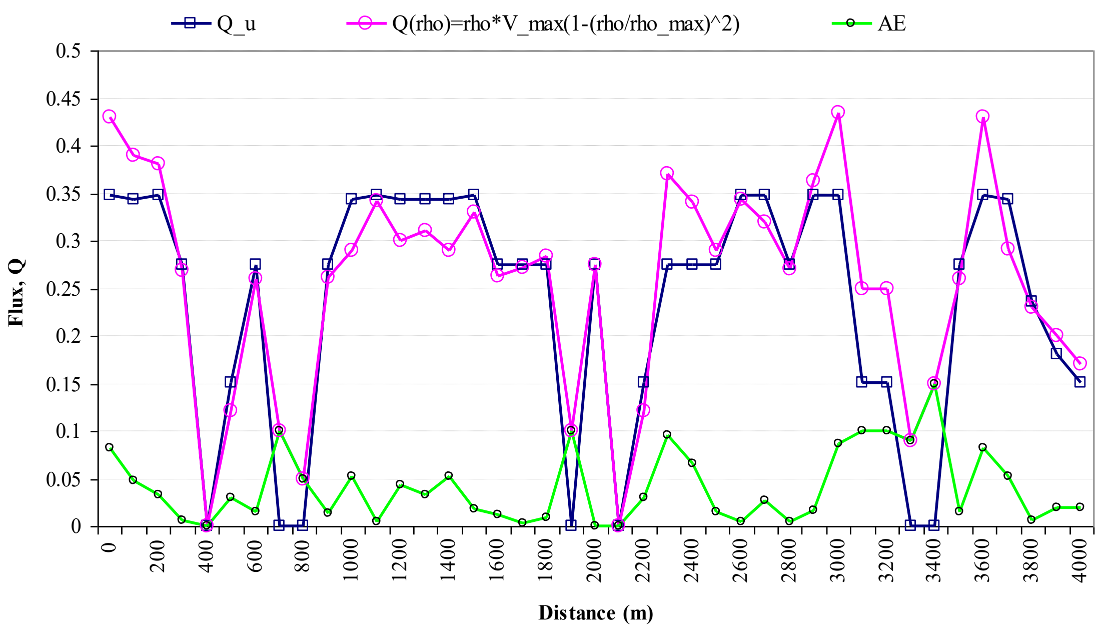

Figure 12: The Velocity Effect of Ramps and Multilane on UP_Lane After 300 Time Steps

Convergence of the Numerical Solution.

The convergence behavior of the numerical solution scheme is shown in the Figure 13. Convergence of the traffic flow model indicates that for the smaller values of time step dt the solution converges in large number of iterations while for increased values of dt the iterations are decreasing. There is exponential decrease in the number of iterations with respect to the time step dt.

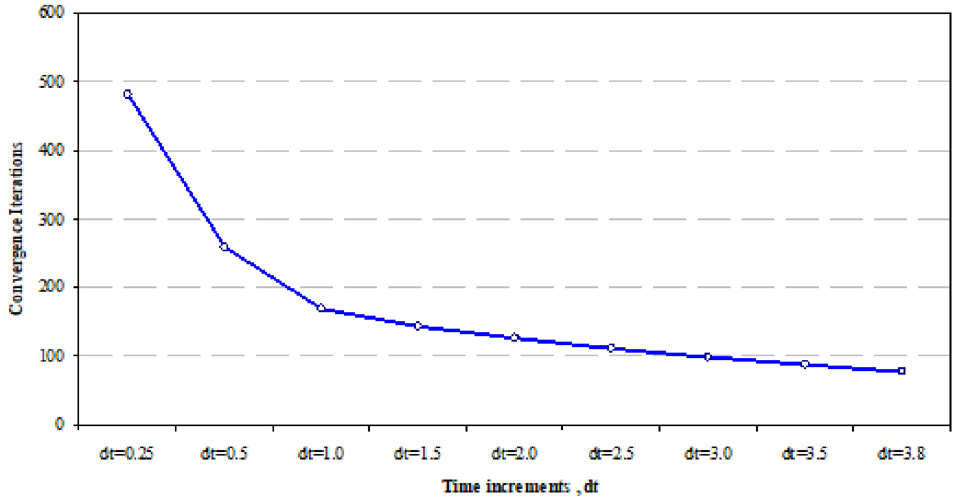

Figure 13: The Convergence Behavior at Different Time Steps

Stability of the Numerical Solution.

The stability of the numerical solution scheme is tested for 1 minute simulation at the different time steps and shown in the Figure 14. It is observed that the solution remains stable for the value of dt<=3.5 and after this value the solution values fluctuate rapidly. It is suggested that the values of dt should lie in the interval [0.25 1.5] for better approximation.

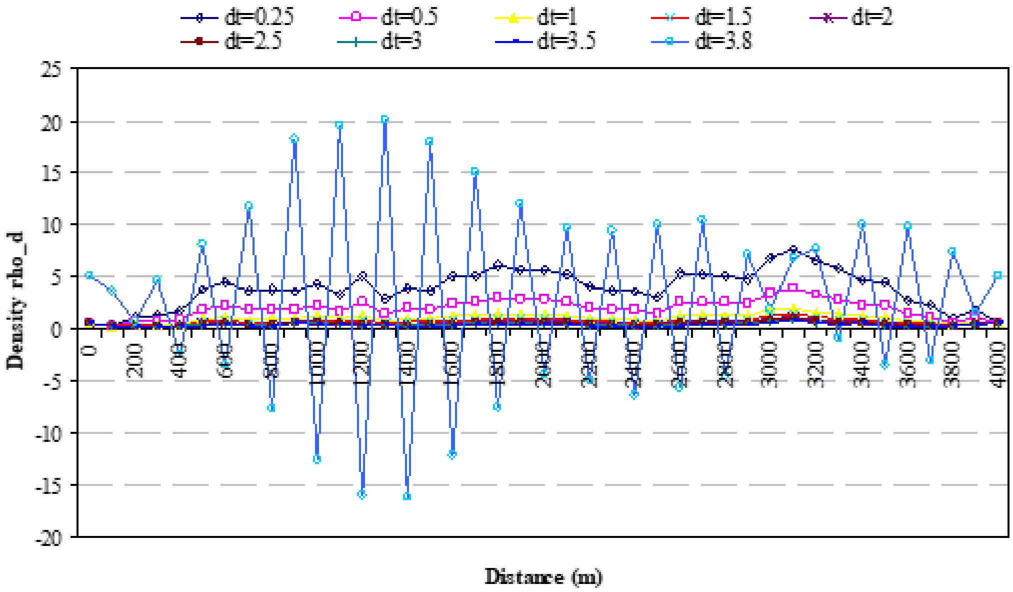

Figure 14: The Stability of the Numerical Solution Scheme

Conclusion

This study presents a macroscopic traffic flow simulation based on the LWR model, incorporating the effects of ramps and multilane roads. Using finite difference methods, simulations in MATLAB successfully modeled traffic density, velocity, and flux across individual and multilane sections. The results revealed significant influences of ramp locations—particularly ramp-on zones—on traffic dynamics, with notable variations in density between left and right lanes. The numerical analysis demonstrated realistic alignment with existing flux models, with minimal discrepancies, and identified stable convergence behavior within specific time step ranges. Overall, the research offers valuable insights into highway traffic flow management, especially in scenarios involving ramps and lane variations. Future work may explore multi-vehicle simulations, extend the model to higher dimensions for path tracing, incorporate finite element methods, evaluate the impact of traffic signals or toll plazas, and analyze systems with more than two lanes to further enhance the model's applicability.

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